

Problem 4

In the following problems, find the limit of the given sequence as $n \rightarrow \infty$.

$$\frac{2^n}{n^2}$$

Solution

Take the limit as $n \rightarrow \infty$, using l'Hôpital's rule where it's appropriate.

$$\begin{aligned}\lim_{n \rightarrow \infty} \frac{2^n}{n^2} &= \lim_{n \rightarrow \infty} \frac{e^{\ln 2^n}}{n^2} \\ &= \lim_{n \rightarrow \infty} \frac{e^{n \ln 2}}{n^2} \\ &\stackrel{\frac{\infty}{\infty}}{=} \lim_{n \rightarrow \infty} \frac{\frac{d}{dn}(e^{n \ln 2})}{\frac{d}{dn}(n^2)} \\ &= \lim_{n \rightarrow \infty} \frac{(e^{n \ln 2}) \frac{d}{dn}(n \ln 2)}{(2n)} \\ &= \lim_{n \rightarrow \infty} \frac{(e^{n \ln 2})(\ln 2)}{2n} \\ &= \frac{\ln 2}{2} \lim_{n \rightarrow \infty} \frac{e^{n \ln 2}}{n} \\ &\stackrel{\frac{\infty}{\infty}}{=} \frac{\ln 2}{2} \lim_{n \rightarrow \infty} \frac{\frac{d}{dn}(e^{n \ln 2})}{\frac{d}{dn}(n)} \\ &= \frac{\ln 2}{2} \lim_{n \rightarrow \infty} \frac{(e^{n \ln 2}) \frac{d}{dn}(n \ln 2)}{(1)} \\ &= \frac{\ln 2}{2} \lim_{n \rightarrow \infty} (e^{n \ln 2})(\ln 2) \\ &= \frac{(\ln 2)^2}{2} \lim_{n \rightarrow \infty} e^{n \ln 2} \\ &= \infty\end{aligned}$$